Assigning uncertainty to input parameters in BEPU analysis: some regulatory insights

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- **This presentation give some insights on the assignment of input probability distributions for BEPU (*Best Estimate Plus Uncertainty*) licensing calculations.**
- **It is rooted on the experience of Technical Staff in Spain's Nuclear Regulatory Authority in the assessment of BEPU methodologies and applications.**

1 BEPU Methodologies

- **Deterministic Safety Analysis (DSA): design of nuclear plants, through the simulation of design basis scenarios (DBS)**
- **BEPU methodologies of DSA:**
  - **Based on realistic computational models (codes) and assumptions**
  - **Includes an uncertainty analysis of the results**
- **There are regulatory acceptance criteria (RAC) on the results of the DBS simulations.**

1 BEPU Methodologies

- **Up to now, the most used BEPU methodologies are:**
  - **Black-box based (i.e. no information about the code is used in the uncertainty propagation.**
  - **Probabilistic (Statistical): uncertainty is modelled with probability (with some exceptions).**
  - **Uncertainty is propagated from inputs and submodels of the code.**
  - **Propagation is performed by pure Monte Carlo i.e. simple random sampling (SRS).**

1 BEPU Methodologies

- **Up to now, the most used BEPU methodologies are (cont.):**

  - **Fulfillment of the RAC is verified through calculation of tolerance intervals via nonparametric Wilks' method (i.e. using order statistics).**

  - **In general no separation between aleatory and epistemic uncertainty, except for the «statistical confidence».**

1 BEPU Methodologies

- **…but there are alternatives:**
  - **Methods based on "propagation from outputs"**
  - **Methods modelling epistemic uncertainty via intervals, or Dempster-Shafer theory, or fuzzy logic…**
  - **Method using other types of sampling, more efficient than SRS. E.g. Latin Hypercube Sampling (LHS).**
  - **Methods using other procedures to calculate tolerance intervals.**

1 BEPU Methodologies

- **Regulatory acceptance criteria (RAC): criteria that must fulfill the safety outputs of DSA calculations**

- **E.g. Y: scalar and continuous safety output, calculated in the simulayion of a DBS.**

L: upper regulatory limit

P: regulatory coverage (or probability) level

C: regulatory confidence level

(P,C): regulatory tolerance level .

Standard is P=C=0.95 ***95/95 criterion***

1 BEPU Methodologies

$$Y_C < L$$

Conservative meth

Uncertainty of Y

$$PR_Y\{Y < L\} \geq P$$

BEPU meth
Cheap-running code

Sample size $\rightarrow \infty$

Uncertainty of DP(Y)

$$PR_{PD(Y)}\{PR_Y\{Y < L\} \geq P\} \geq C$$

BEPU meth
Expensive-running code

2 Input uncertainty

- **Types of input parameters in BEPU analysis of a design basis scenario:**
  - **Initial and boundary conditions**
  - **Properties of the system: material and thermodynamic properties, geometrical and topological parameters.**
  - **Parameters of the numerical model: e.g. time step, size of spatial nodes, etc.**
  - **Operational parameters of the NPP.**
  - **Model parameters**

3 Model parameters

- **The imperfection of physical models is a source of uncertainty:**
  - **Model bias (aka model error, model inadequacy,...): is an imperfectly known quantity (uncertain).**
  - **Model parameters (i.e. quantities involved in model formulation) are uncertain.**
- **Model parameters can be regarded as uncertain input parameters.**

3 Model parameters

- **Model parameters are estimated through inverse methods:**
  - **Point estimates: model calibration.**
  - **Estimates with uncertainty: model uncertainty quantification.**
- **Inverse methods use the discrepancy between model predictions and real data, and propagate it backwards through the model.**

3 Model parameters

- **There are frequentist and Bayesian inverse methods.**
- **Most methods are Bayesian**
  - **Bayesian methods provide regularized solutions (i.e. well-posed).**
- **In last years: many developments and applications of inverse methods for quantification of the uncertainty introduced by physical models in thermohydraulic system codes.**

4 Assigning probability distributions

- **Probability distributions for input parameters, based on real data:**
  - **Nonparametric methods:**
    - ➢ **Empirical distribution: ecdf, histograms,…**
    - ➢ **Kernel methods**
    - ➢ **K-Nearest neighbors methods**
  - **Parametric methods: parametric families are postulated, and the hyperparameters are estimated:**
    - ➢ **Conservatively bounding values of the hyperparameters are assigned.**
    - ➢ **Hyperparameters esimated with uncertainty.**

4 Assigning probability distributions

- **Methods based on maximum entropy principle (MEP):**
  - **Useful when the information is scanty: typically, only the range, mean, variance, and other moments of the variable are known**
  - **Assign the probability distribution which maximizes the entropy.**
  - **Solve a problem of "maximization with constraints". Entropy is maximized, and the constraint is the information.**

4 Assigning probability distributions

- **Methods based on maximum entropy principle (cont.)**

  - **Maximum relative entropy principle is used to include new information: given an estimate p(x) on the pdf, the updated estimate is the function q(x) maximizing the relative entropy of q(x) with respect to p(x) with the constraint introduced by the new information.**

5 Truncating input distributions

- **Truncating the input distribution is not an unusual practice:**
  - **Parametric distributions are truncated to eliminate unphysical values (i.e. physically unreachable) of the magnitude.**
  - **(An alternative is using finite-range distributions e.g. uniform, triangular, trapezoidal,…)**

5 Truncating input distributions

- **Suppose that we discard a portion of the input range i.e. we divide the input range in two subsets:**
  - **The selected region SR having a probability PS**
  - **The discarded region DR having a probability PD**
- **We perform the safety analysis.**
- **A trivial inequality:**

$$PR\{\, Y < L \,\} \;>\; PR\{\, Y < L \mid X \in SR \,\} \cdot PR\{\, X \in SR \,\}$$

5 Truncating input distributions

- **Then,**

$$PR\{\, Y < L \mid X \in SR \,\} \;>\; \frac{P}{PR\,\{\, X \in SR \,\}}$$

and

$$PR\,\{\, X \in SR \,\} \;>\; P$$

implies that

$$PR\,\{\, Y < L \,\} \;>\; P$$

5 Truncating input distributions

- **So a sufficient condition to fulfill the RAC to the regulatory level (P, C) is proving that it is fulfilled**
  - **In a selected input region having a probability content higher than P, and**
  - **With an increased tolerance level (P*, C), with**

$$P^* \equiv \frac{P}{PR\{X \in SR\}}$$

Simple but useful outcome !!

5 Truncating input distributions

- **BEPU calculation with the non-truncated distribution**

is bounded by

- **BEPU calculation with the truncated distribution and an increased coverage level, provided the discarded portion is less than 1-P.**

- **Could an improper input truncation lead to underestimate the input uncertainty?**
- **What if we repeat the BEPU calculation with the non-truncated distribution?**

5 Truncating input distribution

- **E.g. suppose an input parameter with data compatible with a normal distribution N(μ,σ).**

- **For physical reasons, the distribution is truncated in the points $\mu \pm 3\sigma$**

The discarded portion has a probability: $PR(|Z| > 3) = 0.003$

$Z \sim N(0,1)$

5 Truncating input distribution

- **Increased coverage level**

$$P^* = \frac{0.95}{1 - 0.003} = 0.9529$$

- **In nonparametric Wilks' method, the sample size is minimum when the sample maximum is used as upper tolerance level**

$$n_{\min} = \left\lceil \frac{\log(1-C)}{\log P} \right\rceil$$

5 Truncating input distribution

- **For P=C =0.95, n_{min}=59**
- **For P=0.9529, C=0.95, n_{min} =63**

- **When the discarded probability is very low, the Wilks' computational efforts can be equal for the no-truncation case and the truncation case.**

R. Mendizábal, "Some insights on the fulfilment of acceptance criteria by finite mixtures", ANS Best Estimate Plus Uncertainty International Conference (BEPU 2018), BEPU2018-125, Real Collegio, Lucca, Italy, 13-19 May 2018.

6 INPUT PARAMETERS IN TECHNICAL SPECIFICATIONS

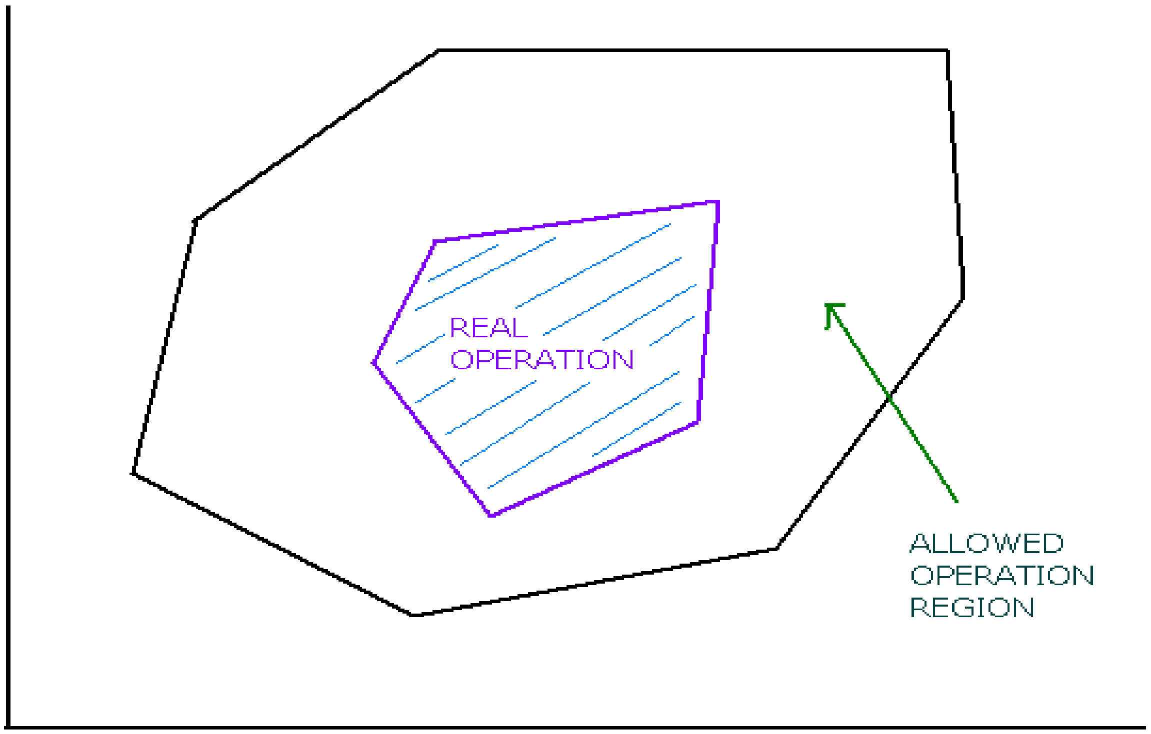
- **A special type of inputs: plant operational parameters which are controlled by Plant Technical Specifications (TS).**
- **There is uncertainty in operational parameters. The TS strongly influences on the uncertainty.**
- **A safety analysis must prove:**

  not only that the «real» operation of the plant is safe

  but

  that the allowed operation of the plant is safe

6 INPUT PARAMETERS IN TECHNICAL SPECIFICATIONS



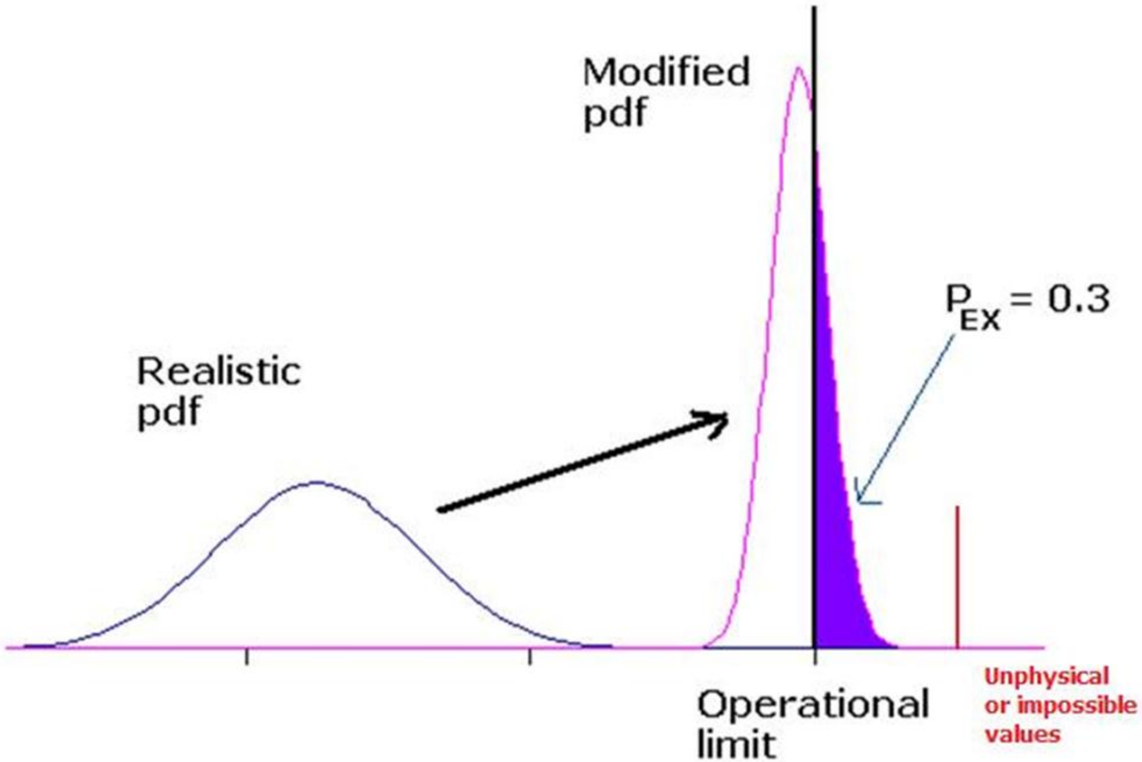
6 INPUT PARAMETERS IN TECHNICAL SPECIFICATIONS

- **Then, in safety analysis, operational parameters which are controlled by TS must be assigned:**
  - **Either fixed values, equal to TS limits (or more conservative)**
  - **Or «fictitious» probability distributions that assign significant probability to the input range around the TS limit**

6 INPUT PARAMETERS IN TECHNICAL SPECIFICATIONS

- **In summary: two possibilities for assigning probability distributions to operational parameters controlled by TS :**
  - **For a BEPU analysis of real operation: realistic distributions, constrained by the TS.**
  - **For a BEPU safety analysis: «fictitious» probability distributions assigning significant probability to the input range around the TS limit**

6 INPUT PARAMETERS IN TECHNICAL SPECIFICATIONS



6 INPUT PARAMETERS IN TECHNICAL SPECIFICATIONS

J.L. Muñoz-Cobo, R. Mendizábal, A. Miquel, C. Berna, A. Escrivá, "Use of the Principles of Maximum Entropy and Maximum Relative Entropy for the Determination of Uncertain Parameter Distributions in Engineering Applications", *Entropy* 2017, 19, 486; doi:10.3390/e19090486 www.mdpi.com/journal/entropy

THANK YOU FOR YOUR ATTENTION !!

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